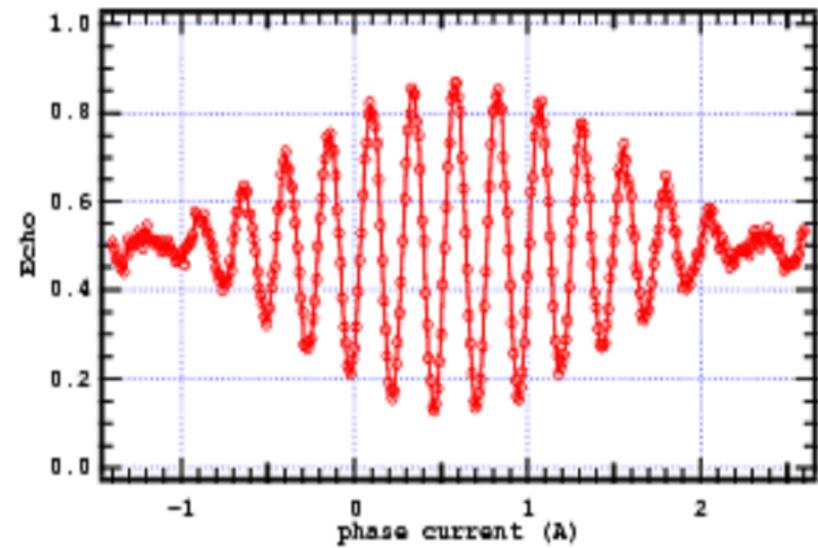
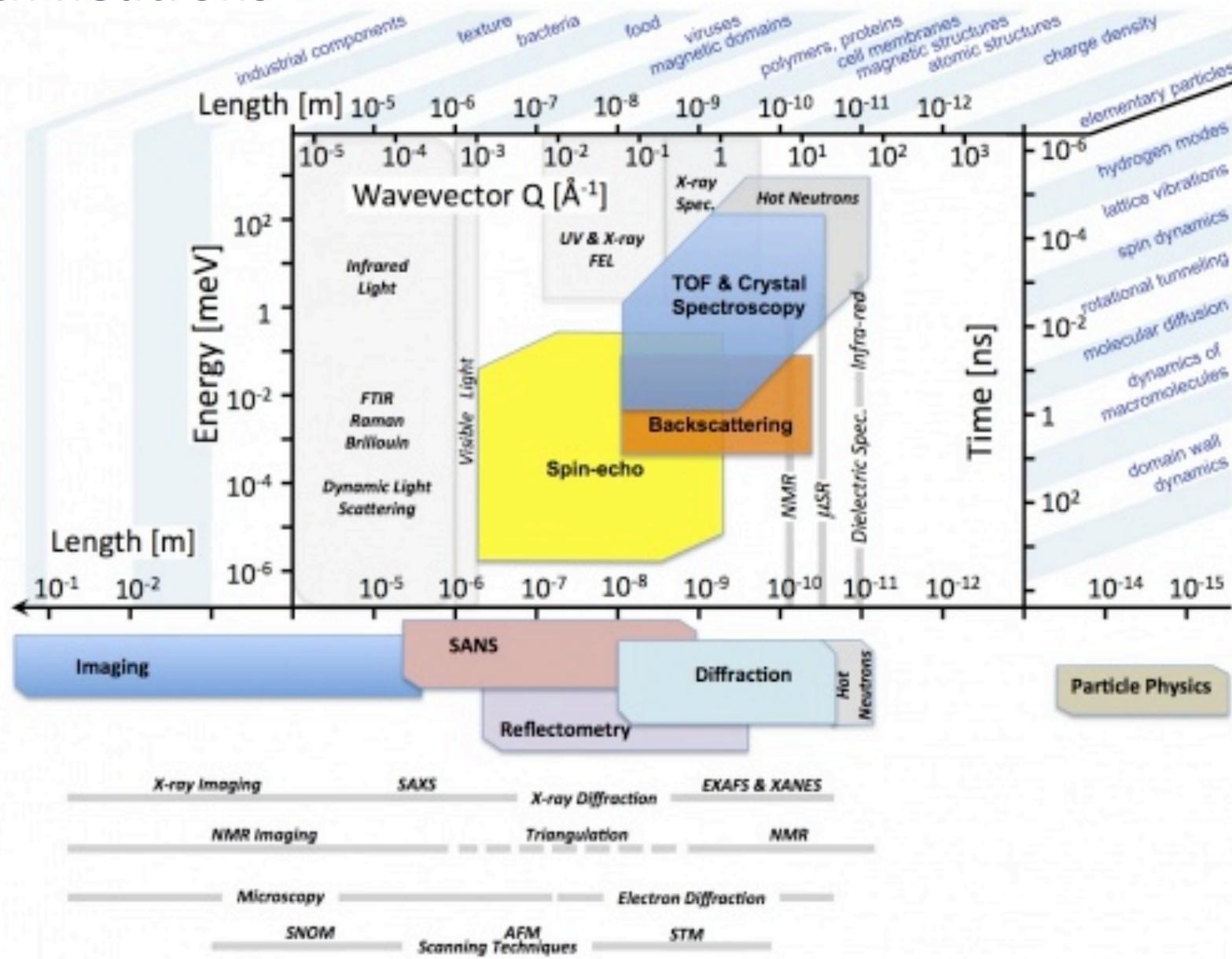


# Neutron Spin Echo Spectroscopy

Jyotsana Lal  
Visiting A. Professor at LSU (LaCNS)



# Exploring the structural and dynamic phase space: with neutrons



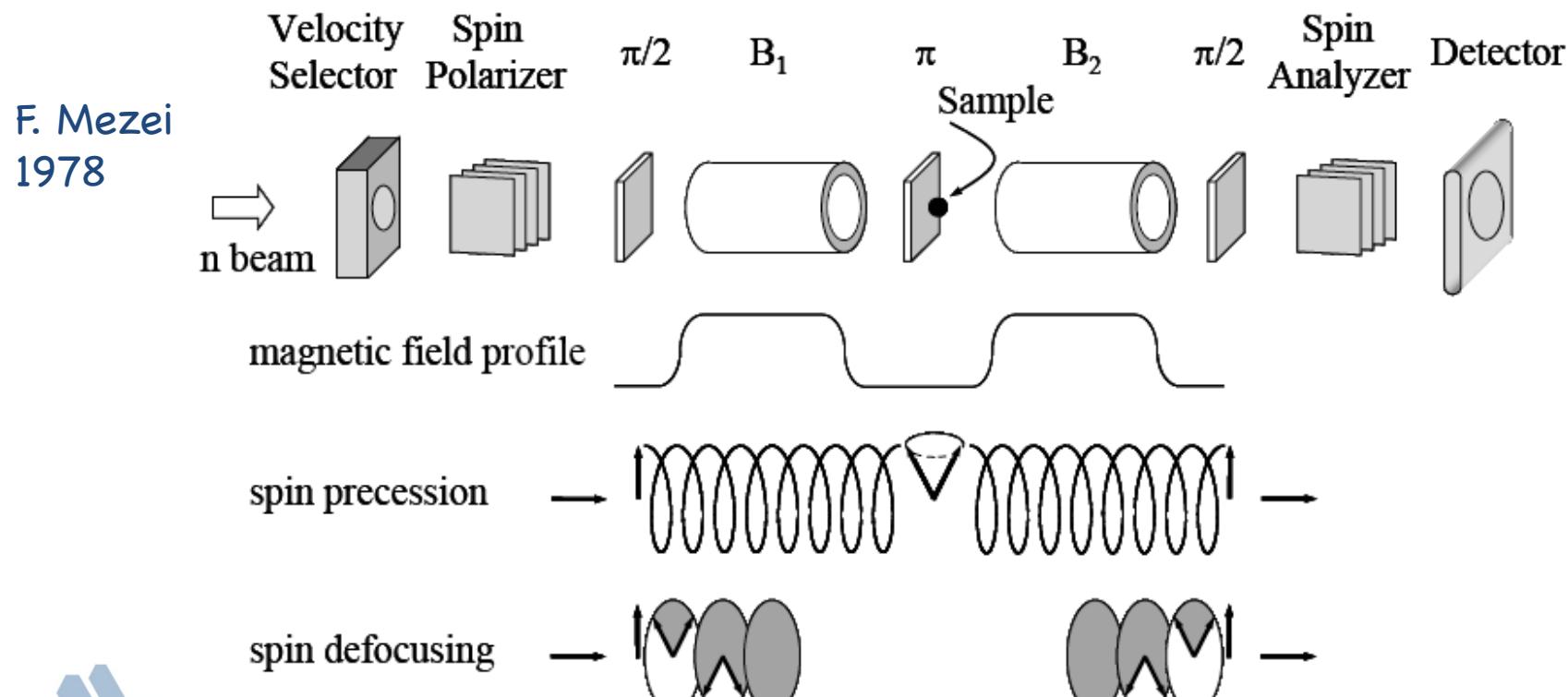
# Detection of slow motions by Neutron Spin echo (NSE)

NSE uses the neutron's spin polarisation to encode the difference in energies between incident and scattered beams. Very high energy resolution.

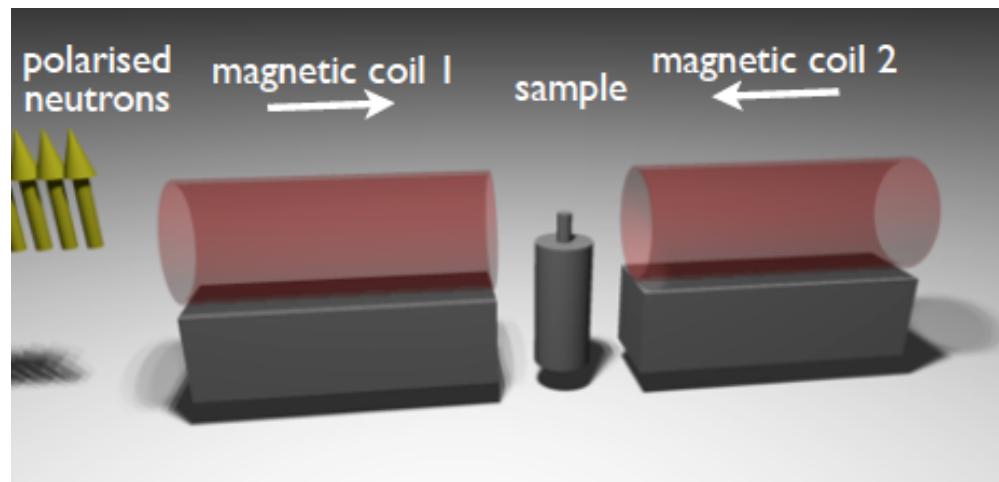
NSE measures the intermediate correlation function in reciprocal space and time  $S(q,t)$ .

NSE spans a time window from  $10^{-12}$  to  $10^{-6}$  s.

## Layout of generic Spin echo Spectrometer



# The measurement principle of neutron spin echo spectroscopy- Classical Description



No strong monochromatisation  
needed

Neutrons are polarised  
perpendicular to magnetic fields

**Elastic Case:** Neutrons  
Perform the same number  
of spin rotations in both  
coils and exit with the  
original polarisation  
(spin echo condition)

**Quasielastic Case:** Time spent  
in the second coil will be slightly  
different, i.e., the original  
polarisation angle is not  
recovered (loss in polarisation)



# The measurement principle of neutron spin echo spectroscopy- Classical Description

Now we will consider how to link the precession to the dynamics in the sample.

Performing a **spin echo experiment** we measure the **polarisation P** with respect to an arbitrarily chosen coordinate x. Px is the projection on this axis and we have to take the average over all precession angles:

$$P_x = \langle \cos \varphi \rangle = \langle \cos(\varphi_{in} - \varphi_{out}) \rangle$$

$$P_x = \langle \cos[\gamma_L \left( \frac{\int \vec{B}_{in} \cdot d\vec{l}}{v_{in}} - \frac{\int \vec{B}_{out} \cdot d\vec{l}}{v_{out}} \right)] \rangle$$

To first order  $\phi$  is proportional to the energy transfer at the sample  $\omega$  with the proportionality constant t (spin echo time).

$$\varphi = t\omega$$

This is the “**fundamental equation**” of classical neutron spin echo.



# The measurement principle of neutron spin echo spectroscopy – Classical description

We consider the “fundamental equation”  $\varphi = t\omega$  and we will calculate  $t$  to first order by Taylor expansion.

Starting point is the energy transfer  $\omega$ :

$$\hbar\omega = \frac{m}{2} [(\bar{v} + \Delta v_{out})^2 - (\bar{v} + \Delta v_{in})^2]$$

Taylor expansion to first order gives:

$$\omega = \frac{m}{\hbar} [\bar{v}\Delta v_{out} - \bar{v}\Delta v_{in}]$$

Now we turn to the phase  $\varphi$ :

$$\varphi = \gamma_L \left[ \frac{\int \vec{B} \cdot d\vec{l}}{\bar{v} + \Delta v_{in}} - \frac{\int \vec{B} \cdot d\vec{l}}{\bar{v} + \Delta v_{out}} \right]$$

Here, Taylor expansion to first order gives:

$$\varphi = \gamma_L \left[ \frac{\int \vec{B} \cdot d\vec{l}}{\bar{v}^2} \Delta v_{out} - \frac{\int \vec{B} \cdot d\vec{l}}{\bar{v}^2} \Delta v_{in} \right]$$

Combining the equations for  $\omega$  and  $\varphi$ , we get:

$$t = \frac{\varphi}{\omega} = \frac{\hbar \gamma_L \int \vec{B} \cdot d\vec{l}}{m \bar{v}^3} = \frac{m^2 \gamma_L \int \vec{B} \cdot d\vec{l}}{2\pi h^2} \lambda^3$$

using de Broglie  $p = mv = \frac{h}{\lambda}$



# The measurement principle of neutron spin echo spectroscopy Classical Description

We return to the equation for the **polarization  $P_x$**

$$P_x = \langle \cos \varphi \rangle = \langle \cos(\omega t) \rangle$$

and use it to prove that we measure the intermediate scattering function.  
In a first step we **write down the average as an integral**

$$P_x(Q, t) = \frac{\int S(Q, \omega) \cos(\omega t) d\omega}{\int S(Q, \omega) d\omega}$$

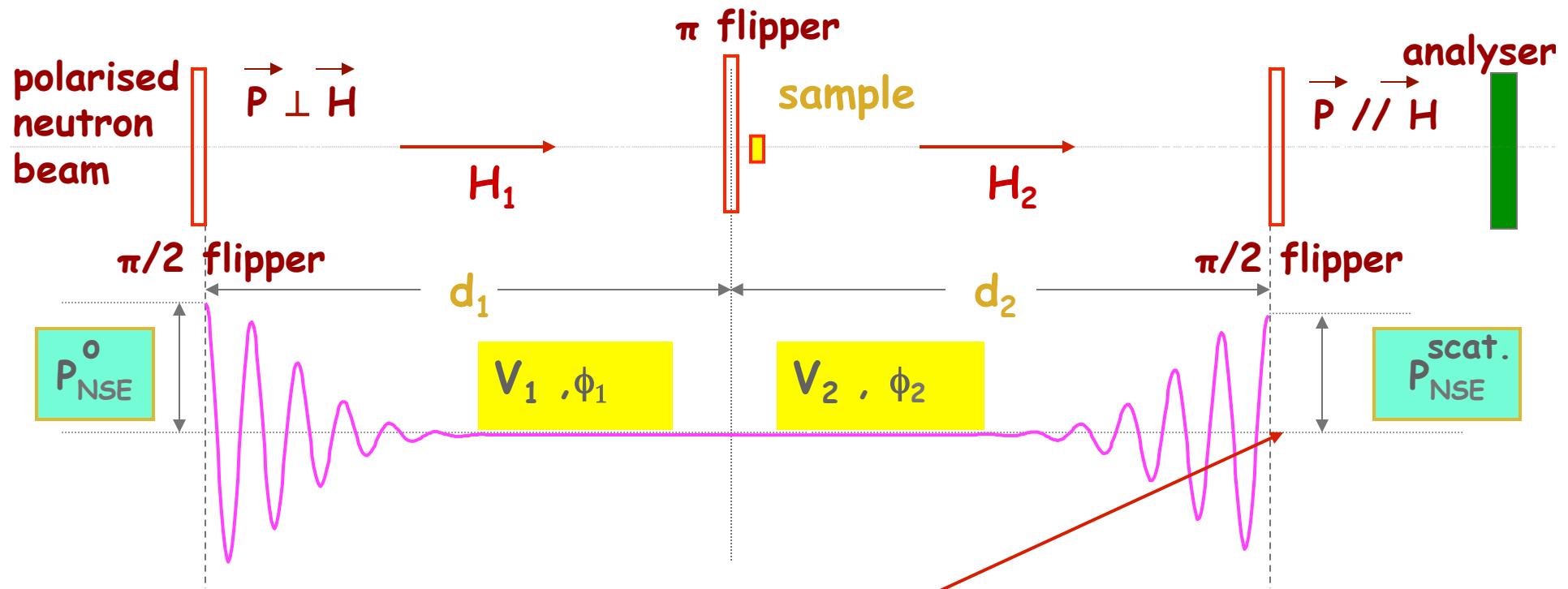
Here, we exploit that the scattering function  $S(Q, \omega)$  is the probability for scattering a neutron with a given momentum and energy transfer.

It turns out that  $P_x$  is the **cosine transform of the  $S(Q, \omega)$** . Thus,  $P_x$  is not strictly equal to the intermediate scattering function, but to the real part only.

$$P_x(Q, t) = \frac{\Re(I(Q, t))}{I(Q, 0)}$$

For most cases this difference is negligible, but this has to be kept in mind.





Total precession angle  $\propto$  energy change of neutrons

$$\Delta\phi = \phi_1 - \phi_2 = \gamma \left[ \frac{d_1 H_1}{v_1} - \frac{d_2 H_2}{v_2} \right] \propto t \Delta \omega$$

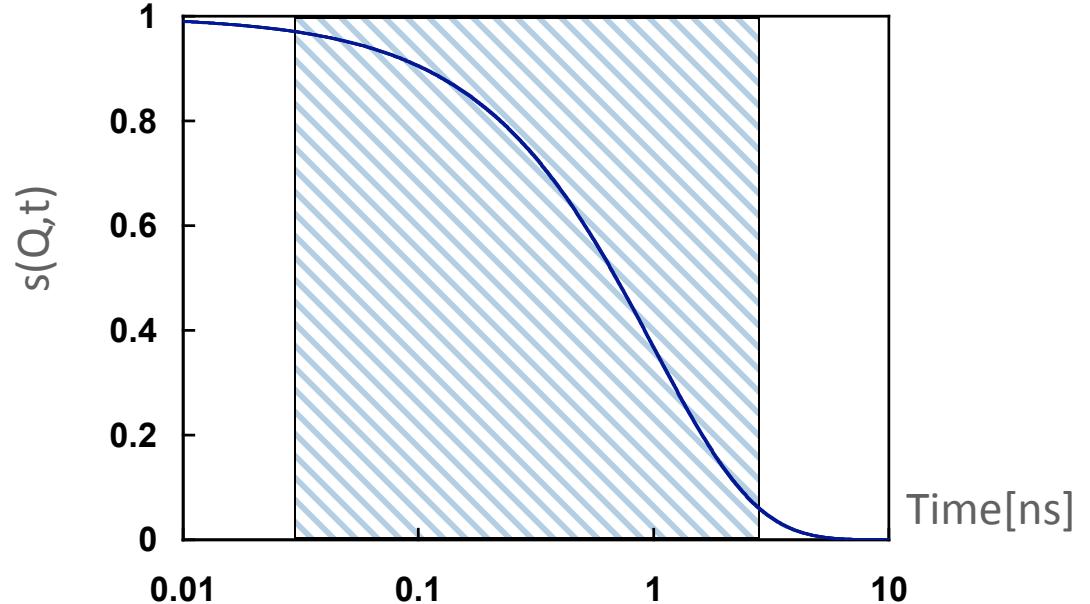


# Intermediate scattering function

$$\frac{P_{NSE}^{sample}(q,t)}{P_{NSE}^{reference}(q,t)} = S(q,t)$$

exponential relaxation :

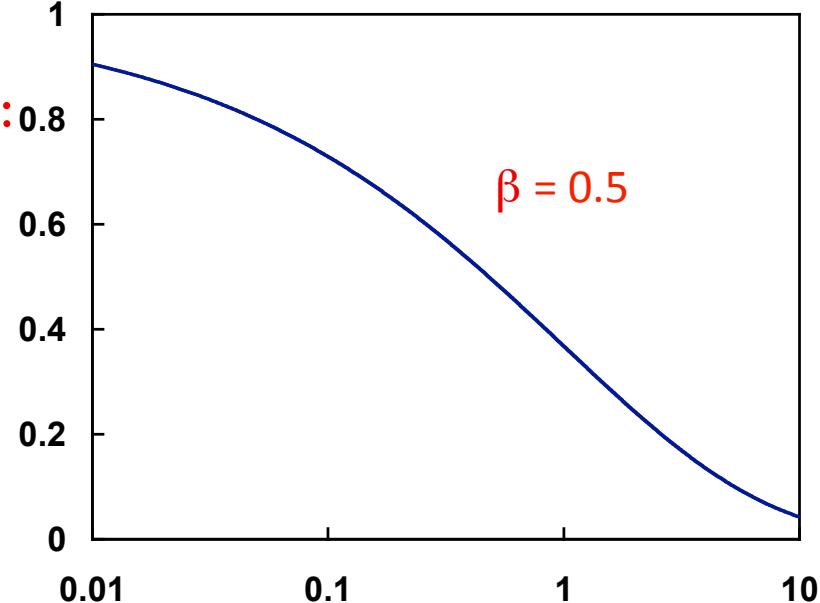
$$s(Q,t) \propto \exp(-t/\tau)$$



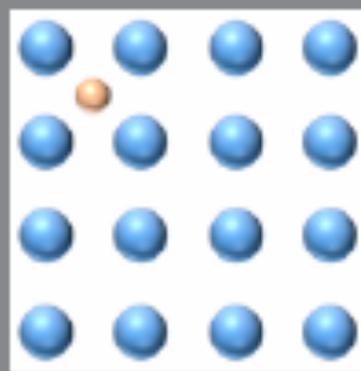
stretched exponential relaxation :

$$s(Q,t) \propto \exp(-(t/\tau)^\beta)$$

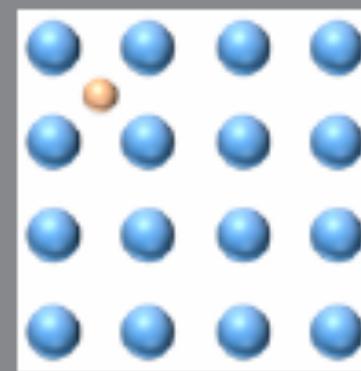
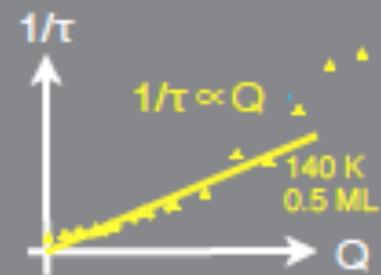
$$\beta < 1$$



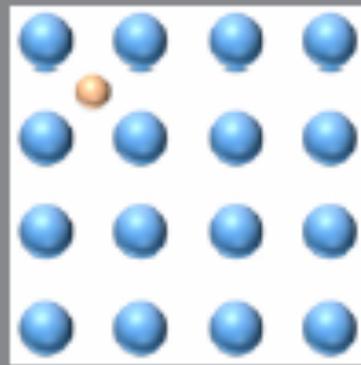
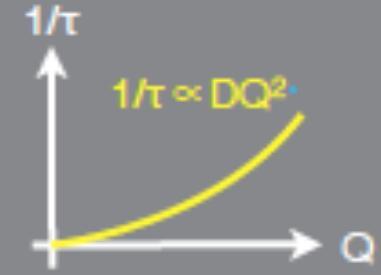
# Exploit the Q dependence of tau or Gamma



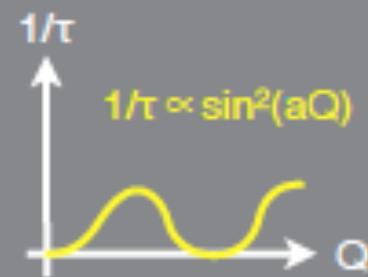
Ballistic Diffusion



Brownian Diffusion



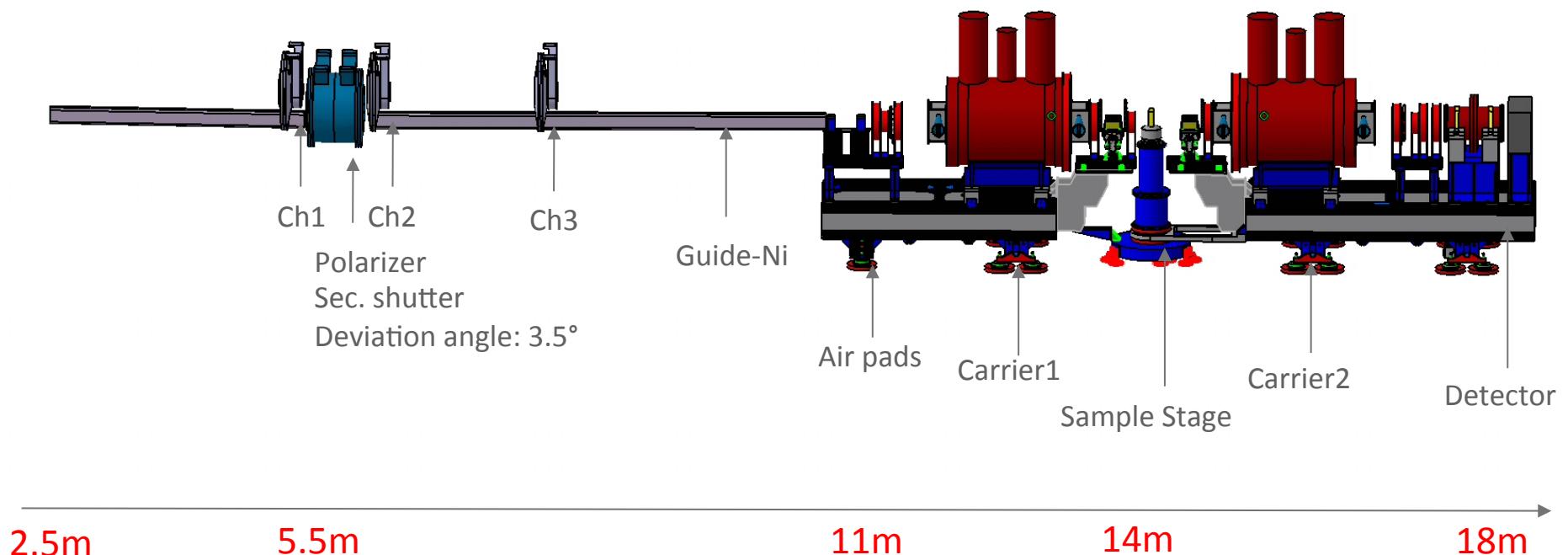
Jump Diffusion



# Neutron spin echo spectrometer

## NSE measures $S(q,t)$

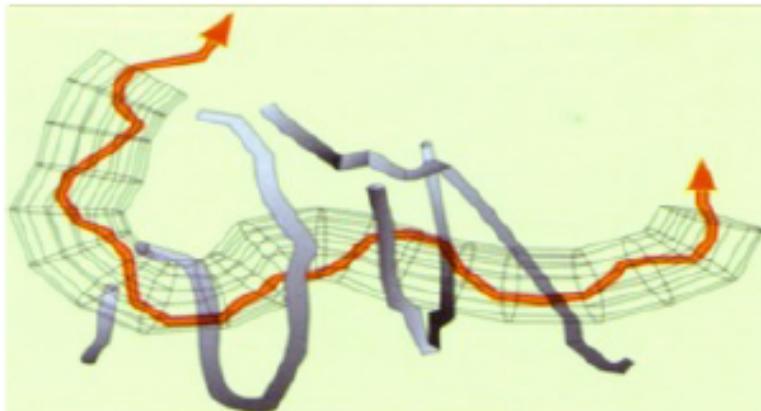
Detection of slow motions  $1 \text{ ps} < t < 1 \mu\text{s}$



SNS

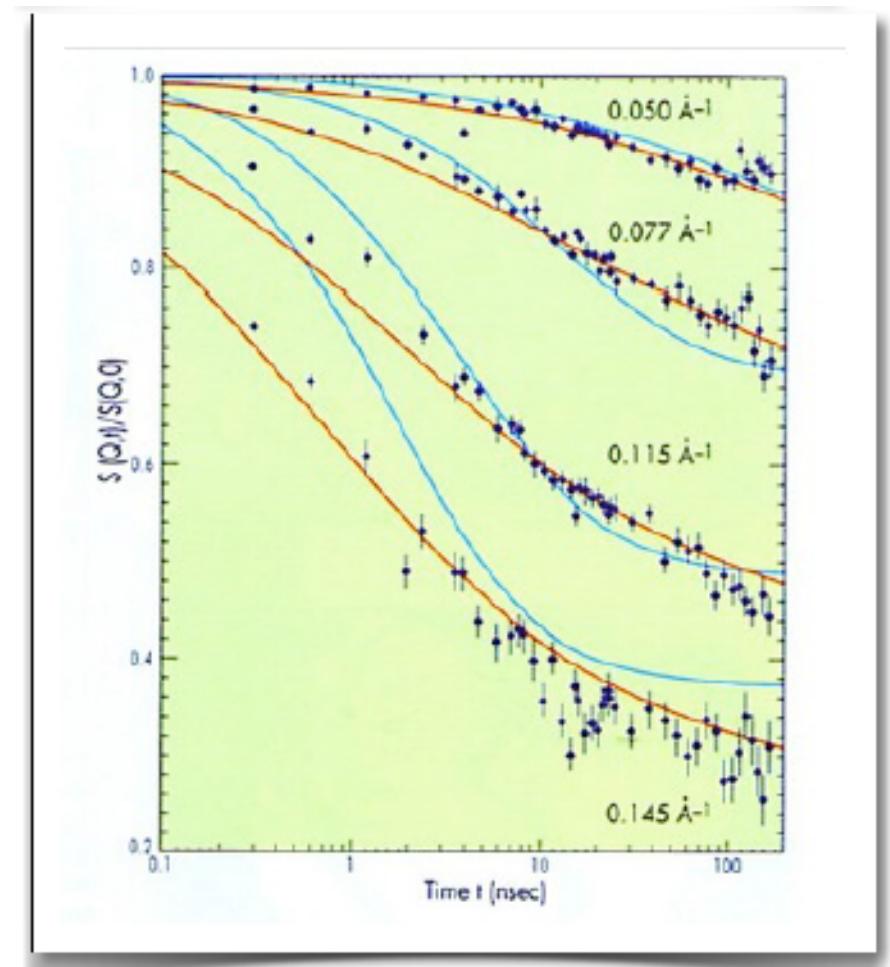
# Examples of NSE studies

## Reptation in polyethylene



de Gennes formulated the reptation hypothesis in which a chain is confined within a “tube” constraining lateral diffusion - although several other models have also been proposed

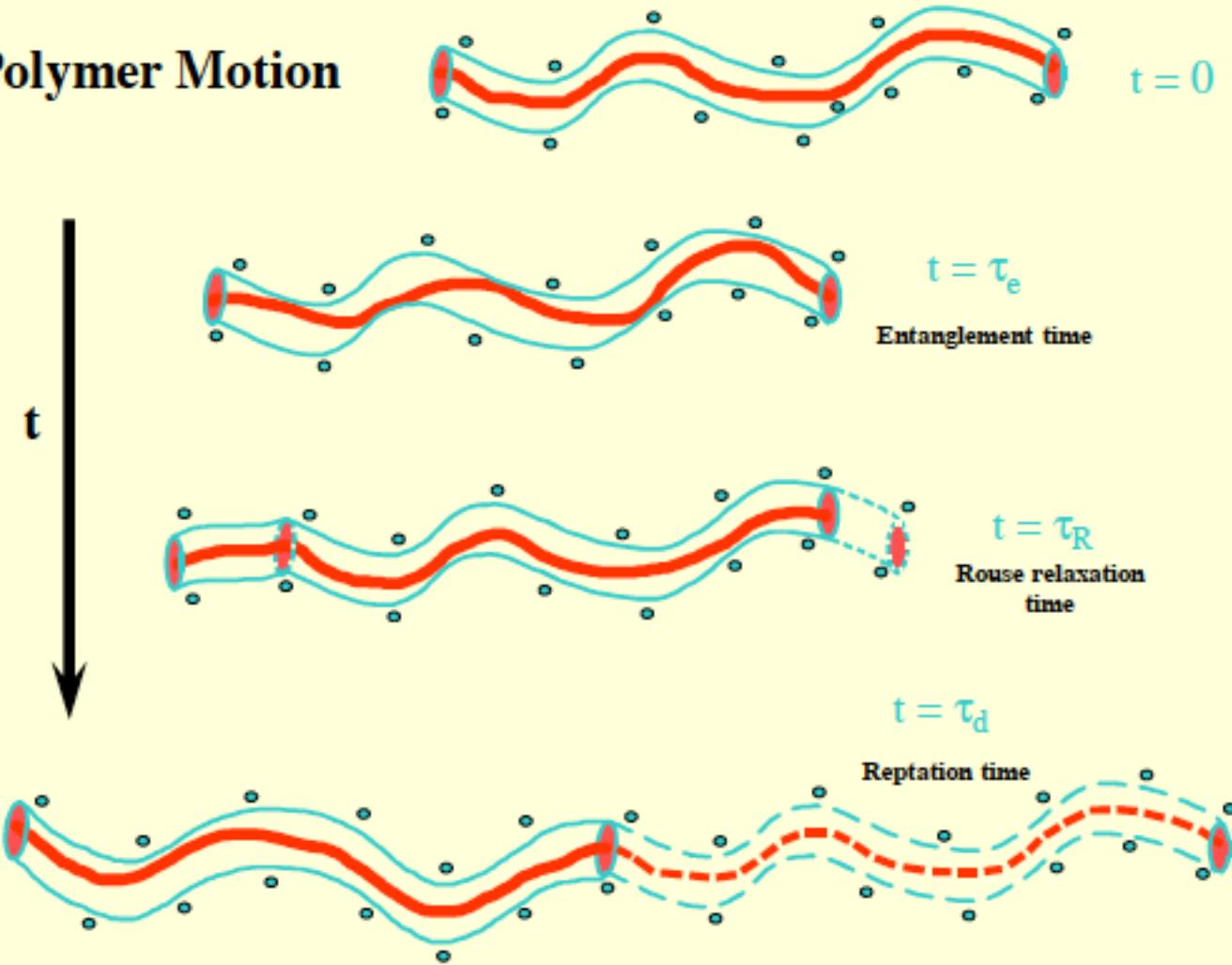
The measurements on IN15 are in agreement with the reptation model. Fits to the model can be made with one free parameter, the tube diameter, which is estimated to be 45Å



Schleger et al, Phys Rev Lett 81, 124 (1998)



## Polymer Motion



$$\Gamma \sim q^4$$

At larger distances  
slow creep or reptation



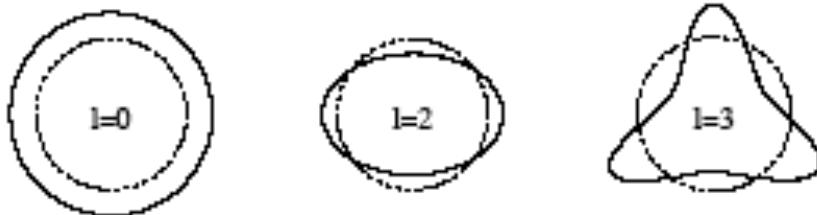
# Undulations of a planar membrane

(Papoulier & de Gennes 1969)

$$\langle h_q(t)h_{-q}(t) \rangle = \langle h_q h_{-q} \rangle e^{-\omega_q t} \quad \left\{ \begin{array}{l} \langle h_q h_{-q} \rangle = \frac{k_B T}{\kappa q^4} \\ \omega_q = \frac{\kappa}{4\eta} q^3 \end{array} \right.$$

# Undulations of a sphere

(Schneider, Jenkins & Webb, 1984. Milner & Safran 1987)



$$r(\Omega) = R[1 + \sum_{l,m} u_{l,m} Y_{l,m}(\Omega)]$$

$$\langle u_{l,m}(t)u_{l,m}(0) \rangle = \langle |u_{l,m}|^2 \rangle e^{-\omega_{l,m} t} \quad \left\{ \begin{array}{l} \langle |u_{l,m}|^2 \rangle = \frac{k_B T}{\kappa} \frac{1}{(l+2)(l-1)[l(l+1)-\gamma]} \\ \omega_{l,m} = \frac{\kappa}{\eta R^3} \frac{(l+2)(l-1)[l(l+1)-\gamma]l(l+1)}{(2l+1)(2l^2+2l-1)} \end{array} \right.$$

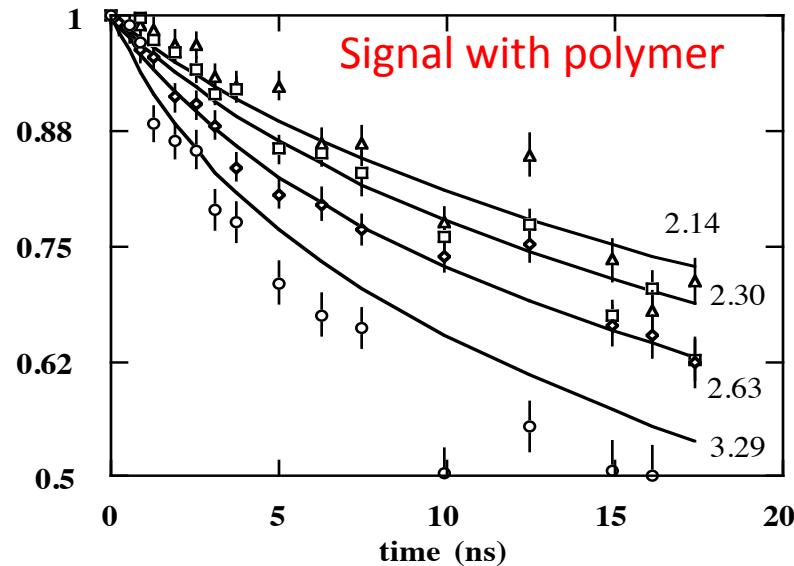
$$\omega_{l,m} \leftrightarrow \omega_\sim$$

$$I(Q, t) = e^{-DQ^2 t} V_s^2 (\Delta\rho)^2 \left[ f_0(Qr_0) + \sum_{l>1} \frac{2l+1}{4\pi r_0^2} f_l(Qr_0) \langle a_l(0) a_l(t) \rangle \right]$$



# Dynamics-Droplet Microemulsions

$I(q,t)/I(q,0)$



Rough analysis

- 1) Safran -Milner
- 2) Polydispersity

Dynamics of the mode  $l=2$   
is slowed down with polymer inside  $\tau_2$

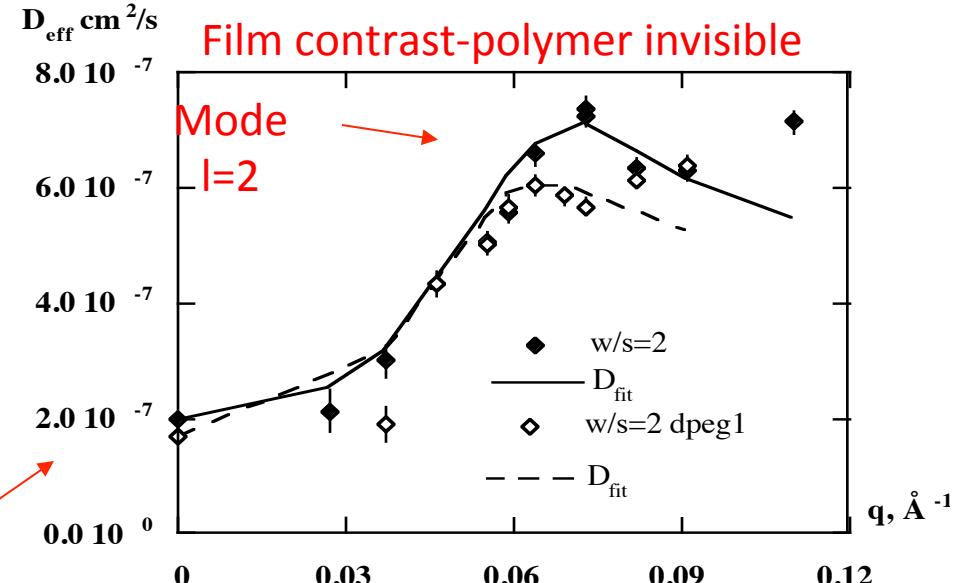
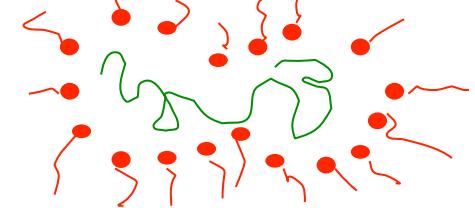
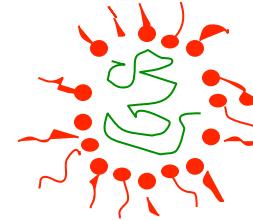
Dissipation is mainly viscous  $\eta$

$K, U_2$  constant



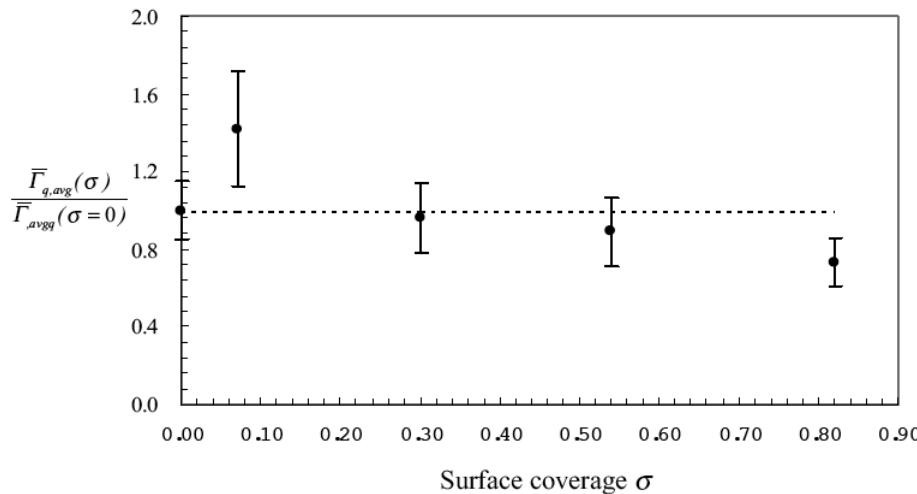
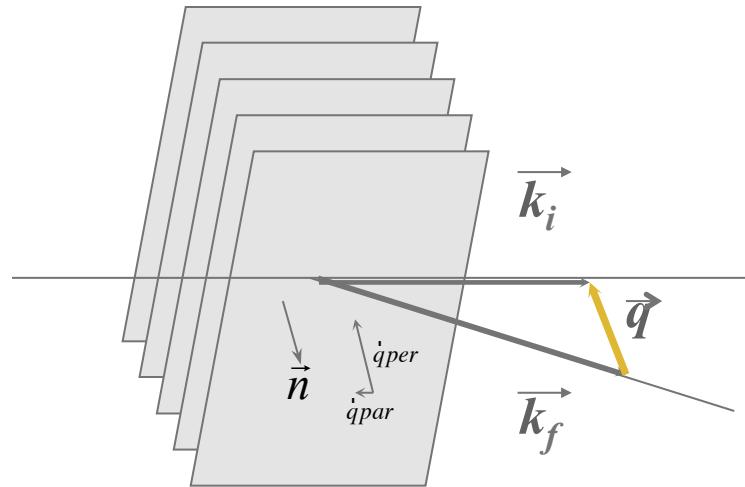
J. Lal, B. Farago and L. Auvray, 1994 MRS Fall Meeting Symposium  
Proceedings-Dynamics in Small Confining Systems II vol. 366, pgs. 427-438

Deformation modes



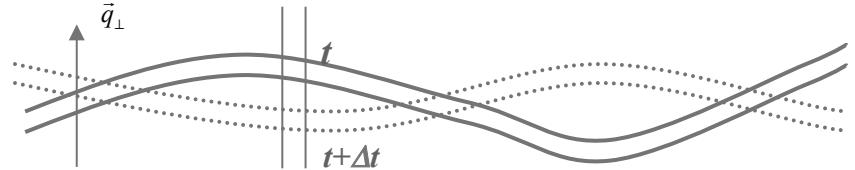
# Hydrophobically Modified Polymer Confined in Surfactant Bilayers

Yang, Lal, Mihailescu, Monkenbusch, Richter, Huang, Kohn, Russel, and Prud'homme

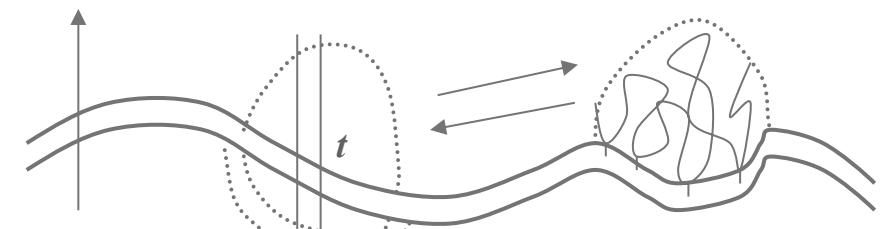


Low coverage “speeding up”

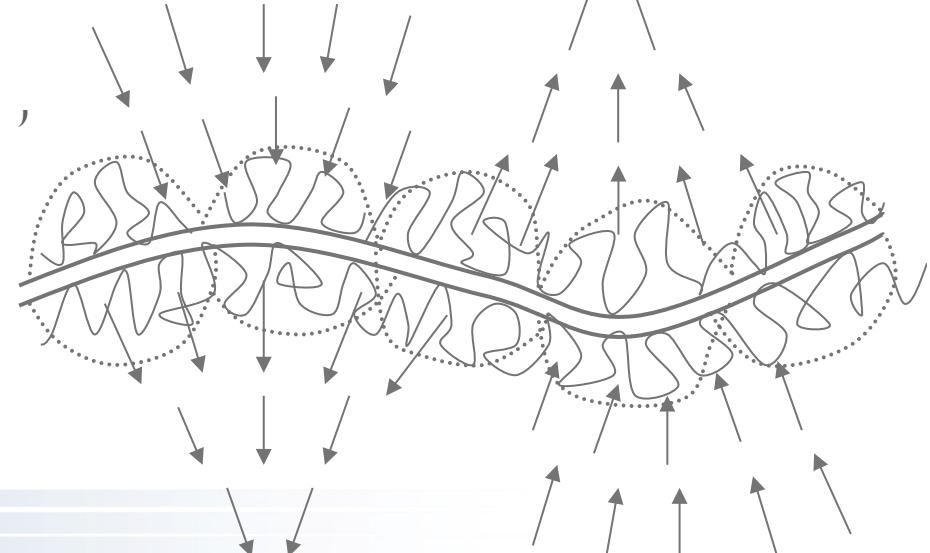
(a) due to membrane being stiffer



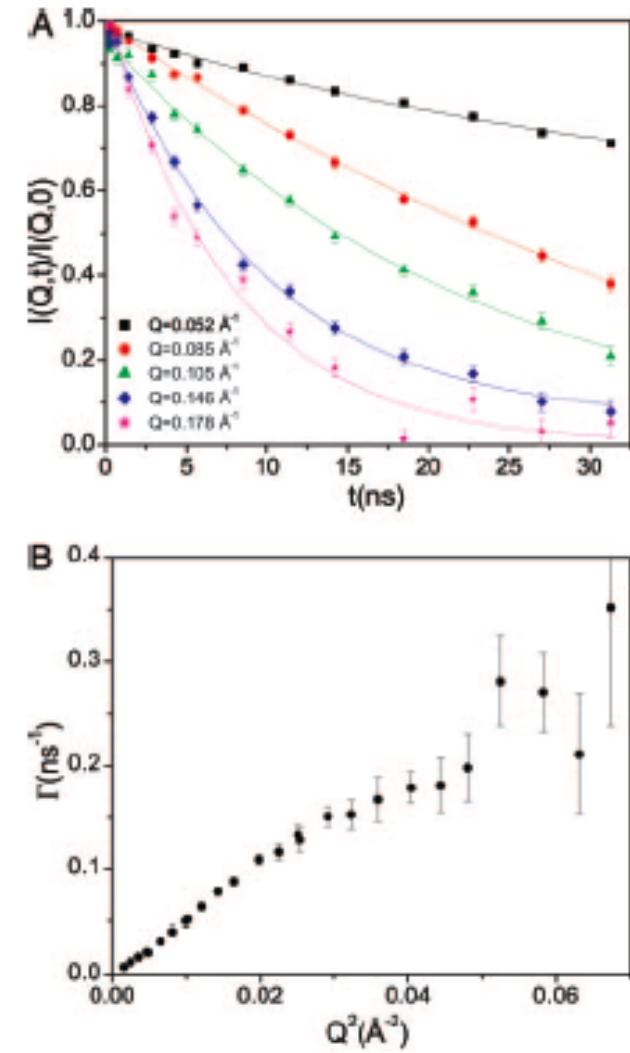
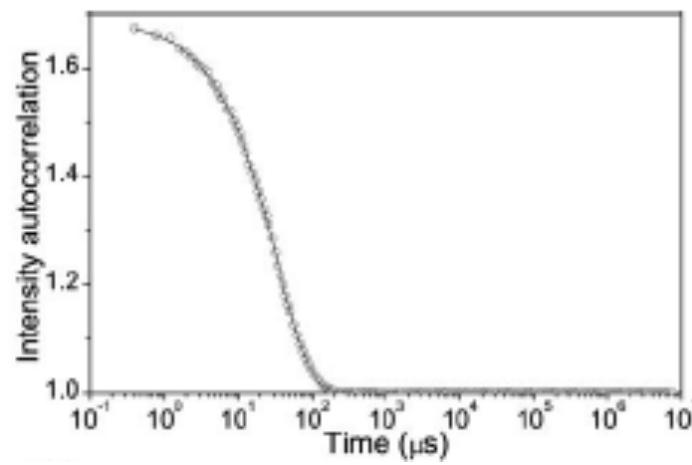
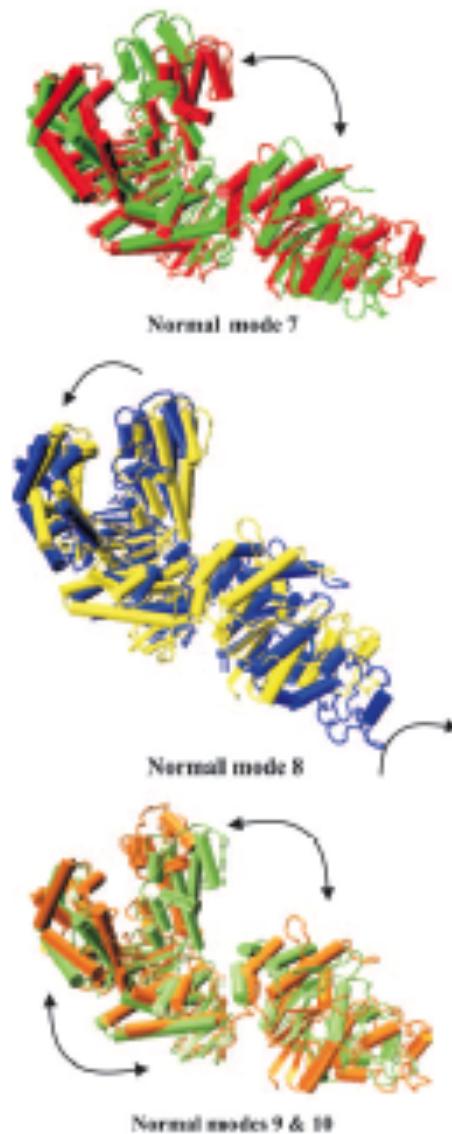
(b) coupling between lateral polymer diffusion and relaxation of undulations



High coverage “slowing down”



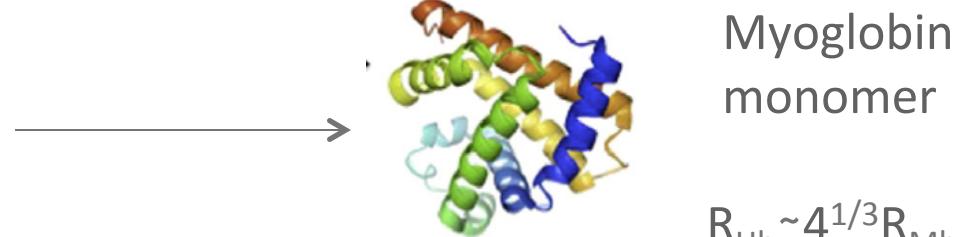
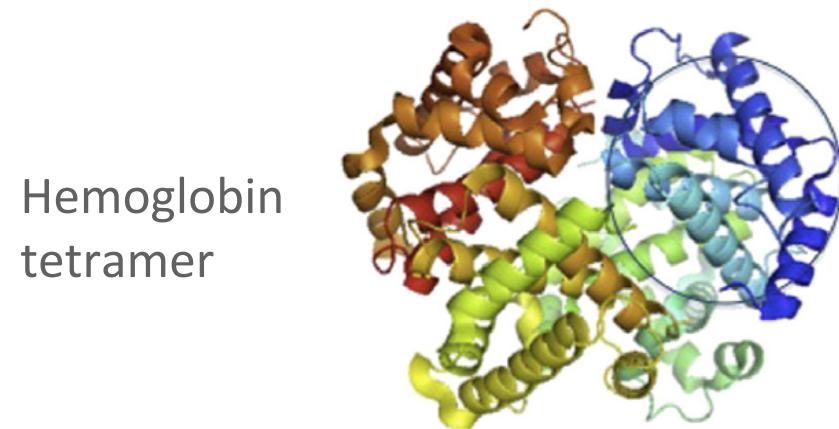
# Scattering techniques for dynamics - NSE & DLS



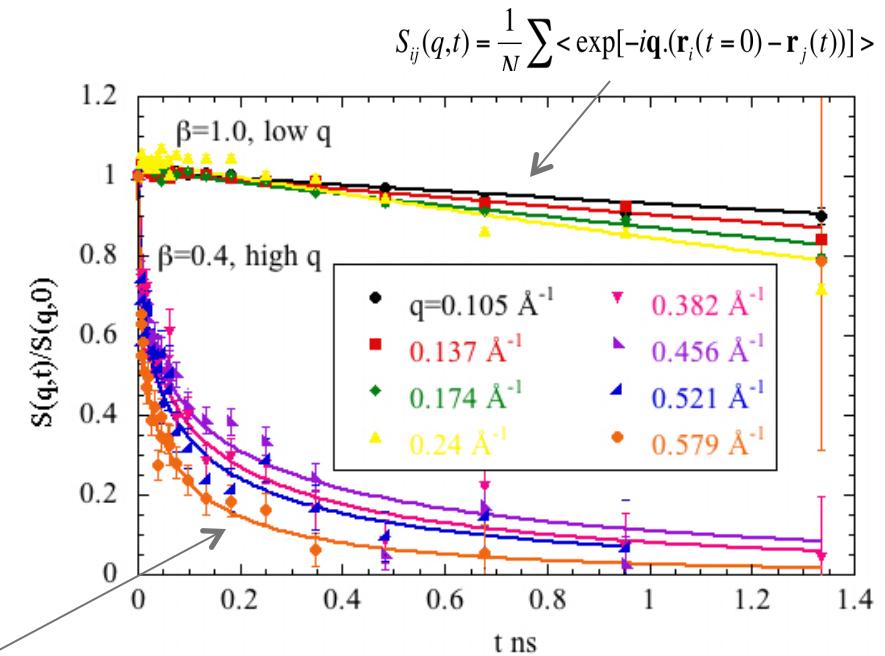
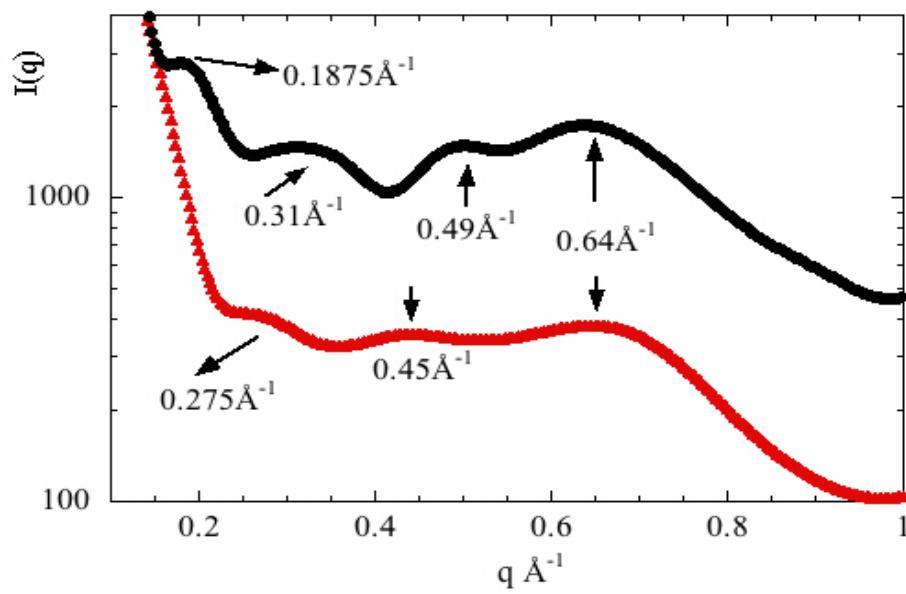
Coupled protein domain motion Taq polymerase-D. Callaway



# NSE study of homologous proteins



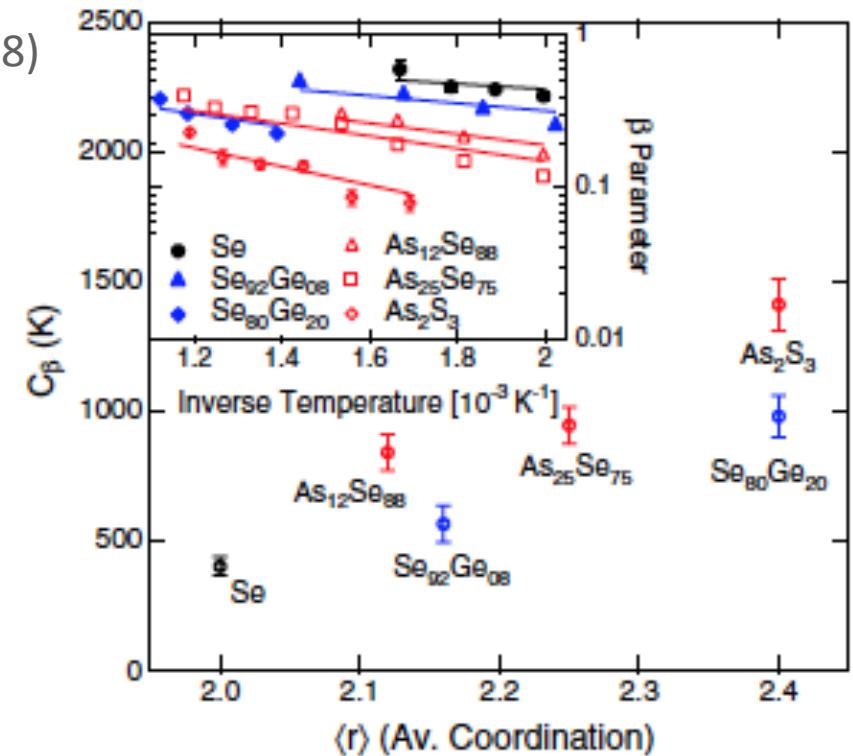
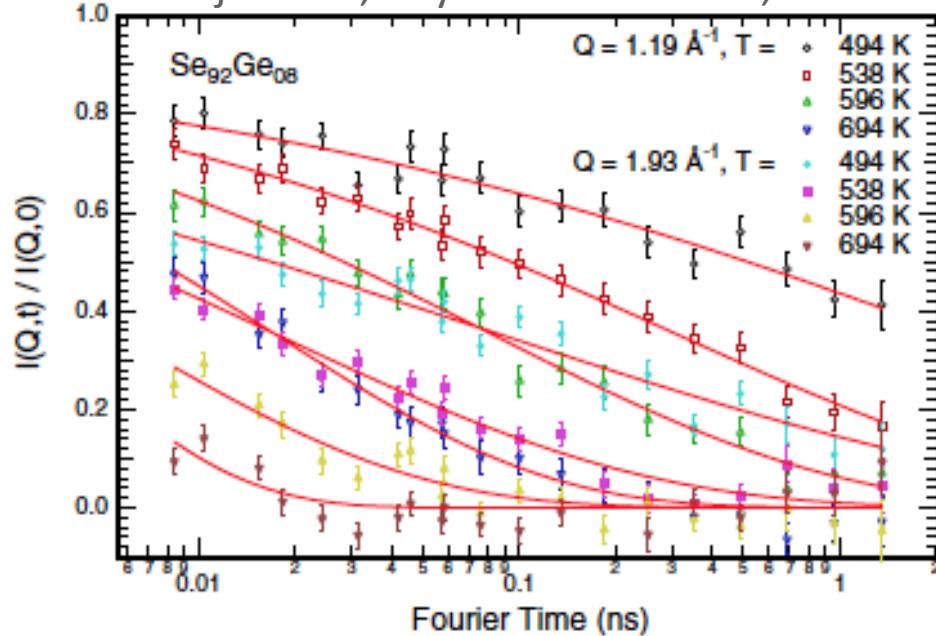
$$R_{Hb} \sim 4^{1/3} R_{Mb}$$



J. Lal et al J. Mol. Bio. 2008

# Dynamics of Glasses

Bermejo et al., Phys. Rev. Lett. 100, 245902 (2008)



$\text{Ge}_x\text{As}_y\text{Se}_{1-x-y}$  is a prime example of a network glass

Normal liquids dynamics show a thermal activation according to  $\exp[-kT/E_a]$

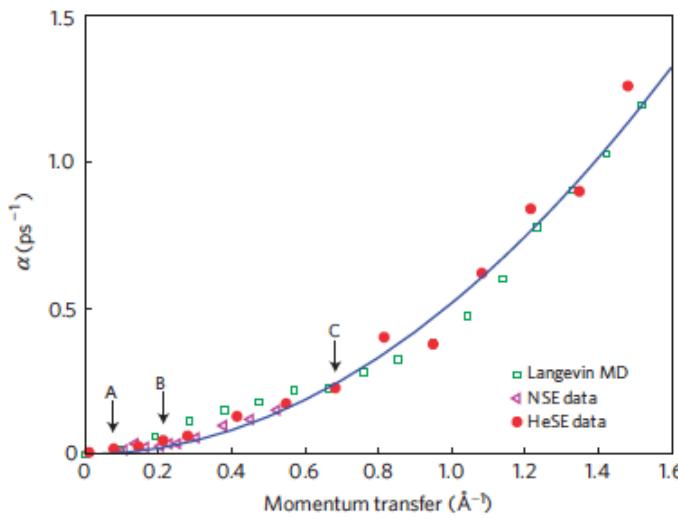
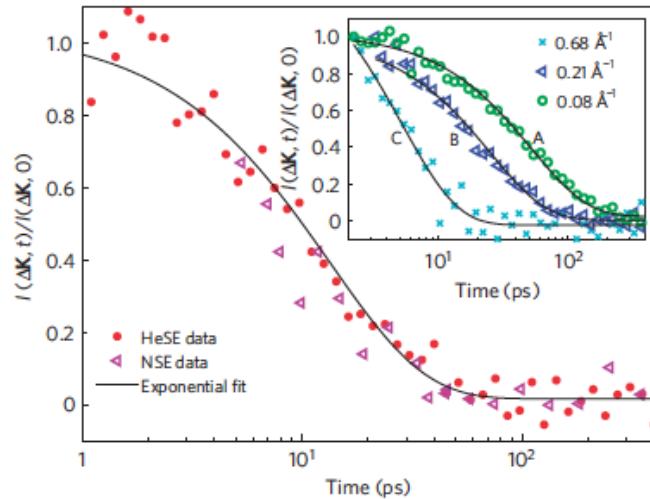
Dynamics of glasses close to the transition temperature, however, show sometimes strong deviations from exponential behaviour

With measurements on IN11 it was possible to see this effect even far away from the glass transition and the dynamics were linked to the average co-ordination number  $\langle r \rangle$



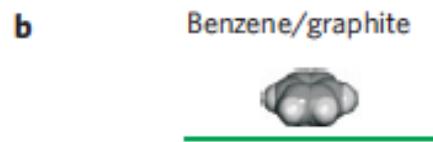
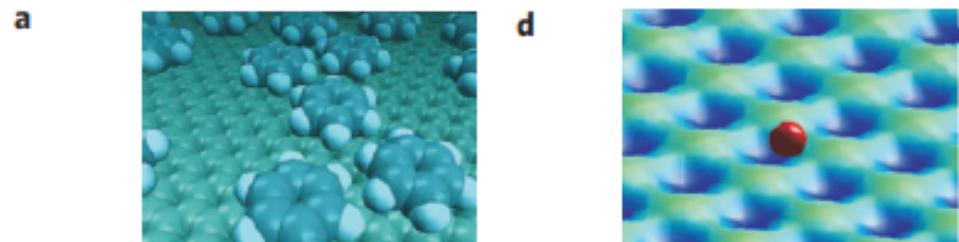
$$\langle r \rangle = 4x + 3y + 2(1-x-y)$$

# Measurement of single-molecule frictional dissipation benzene on a graphite surface

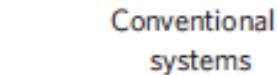
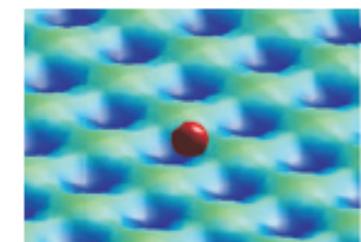


Dynamic friction can be determined.

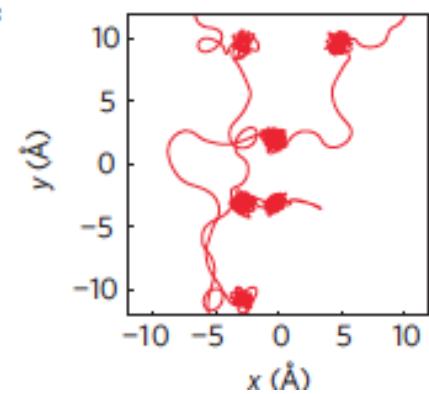
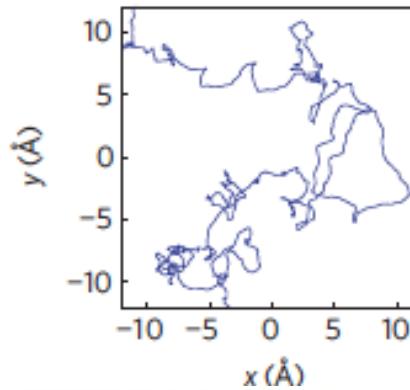
NSE - and its surface equivalent helium spin-echo – see “perfect” molecular Brownian diffusion.



Unrestricted,  
continuous  
diffusion regime.  
Quadratic  $\alpha(\Delta K)$

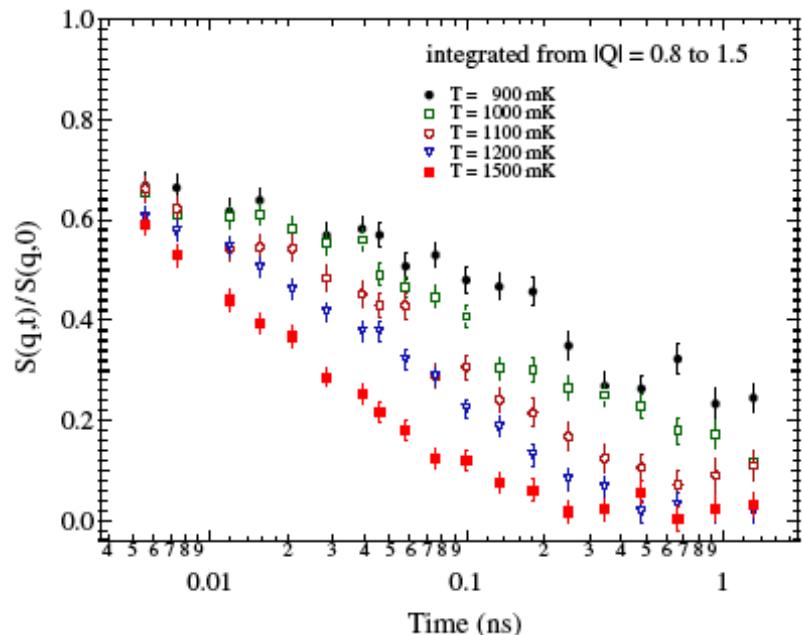


Static barrier  
limited hopping.  
Sinusoidal  $\alpha(\Delta K)$

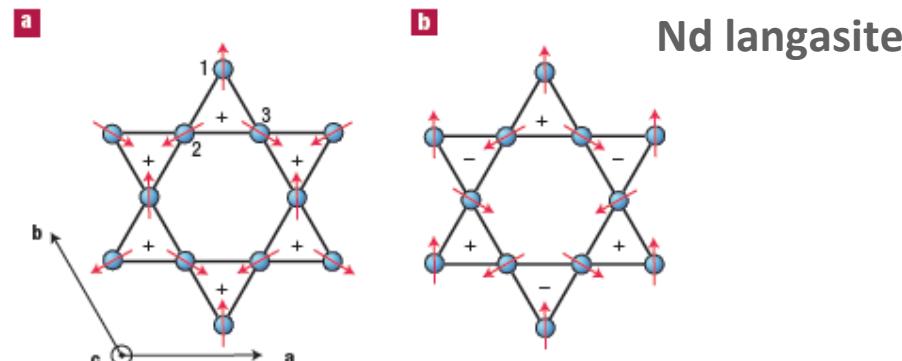


Hedgeland et al., Nature Physics 5, 561 (2009)

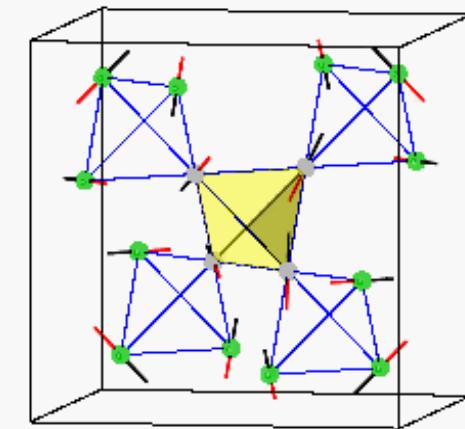
# Dynamics of Frustrated Magnets



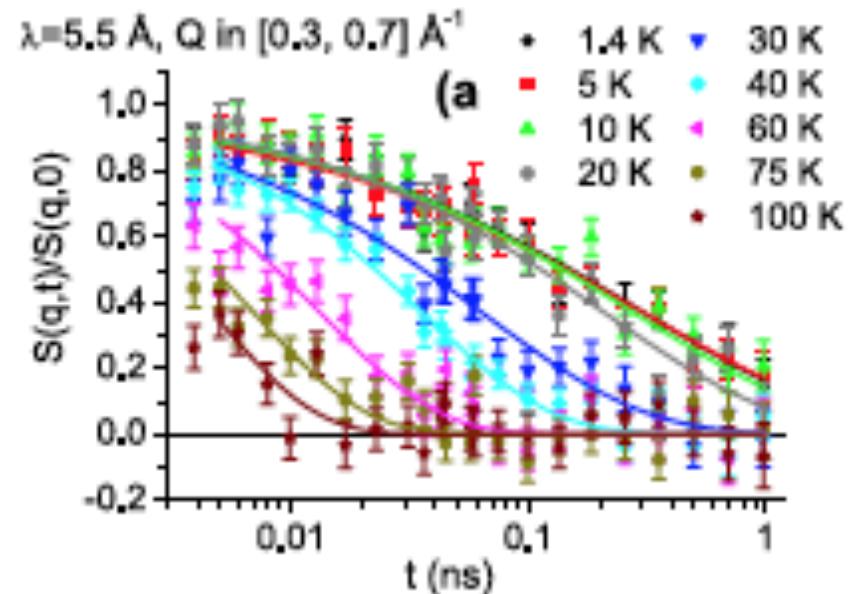
J. of Phys.-Cond. Matter 16 (11) 2004.



D. Grohol et al., Nature Materials 4, 323 (2005)

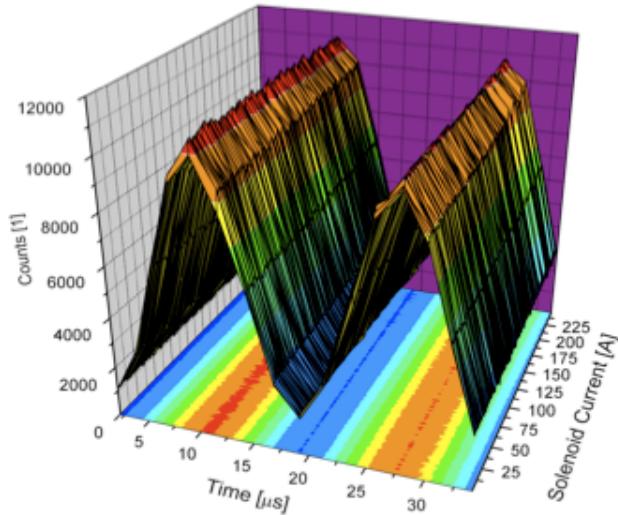
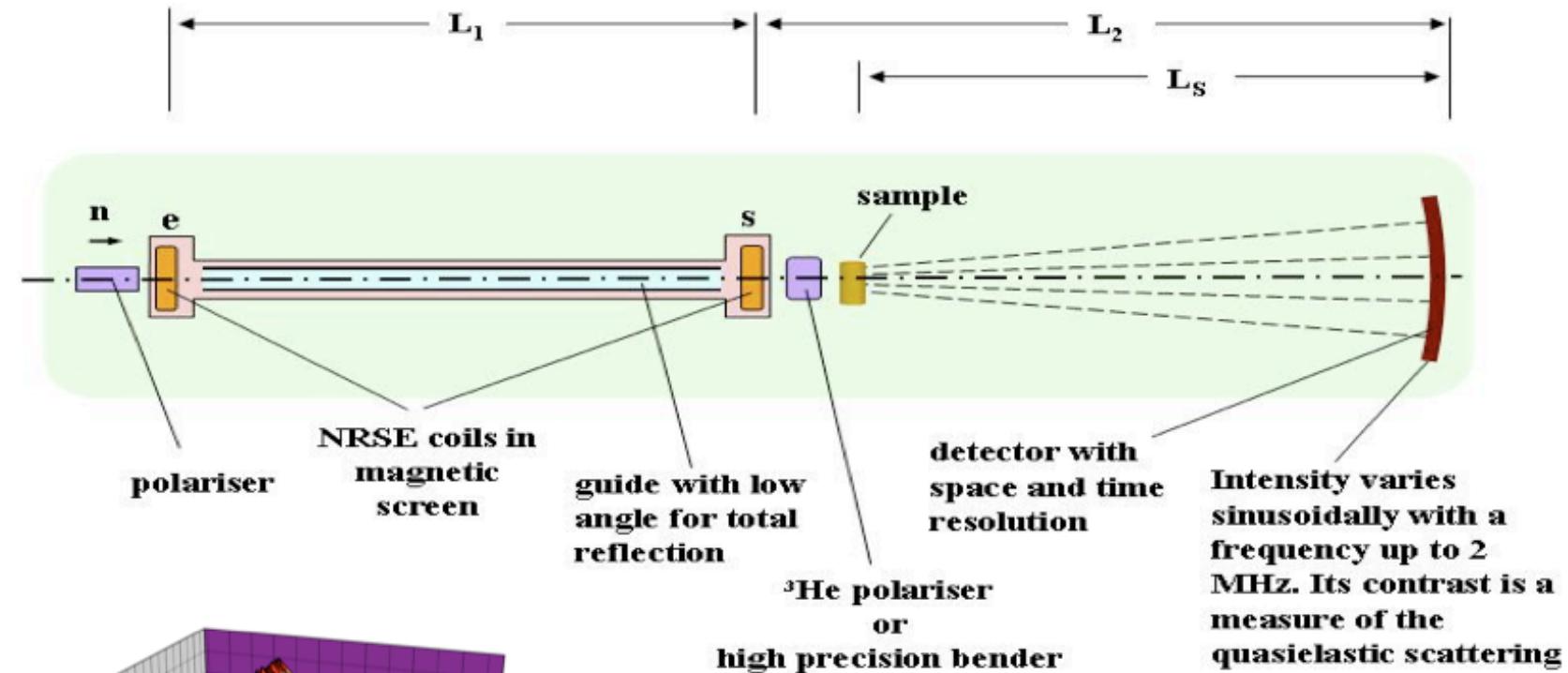


**Gd<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>:** The normalized intermediate scattering function at several different temperatures around the 1 K transition.



V. Simonet et al., PRL 100, 237204 (2008)

# MISANS (Modulated Intensity Small Angle Scattering)



R. Gähler et al., Physica B 180-181 (1992) 899

$$I(t_D) = \frac{I_0}{2} (1 + \cos[2(\omega_B - \omega_A)t_D])$$

M. Bleuel , JL et al , First tests of a MIEZE, 2006.  
<http://www.sns.gov/research/highlights/MISANS/>  
R. Georgii, JL et al *Applied Physics Letters* 2011.

